MAGMA **package** Weight1

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Katz forms of weight one

Katz modular forms over $\mathbb{F}_p$

- are defined algebro-geometrically,
- can be represented by a q-expansion in $\overline{\mathbb{F}}_p[[q]]$,
- of weight ≥ 2 are precisely reductions mod p of classical elliptic modular forms (of the same level and weight)
- of weight 1 are richer than the reductions.

Katz eigenforms of weight 1 over $\mathbb{F}_p$ are characterized by the fact that the attached Galois representation $\rho : \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \to \mathrm{GL}_2(\overline{\mathbb{F}}_p)$ is unramified at p.

They also explain some interesting properties of certain Hecke algebras (e.g. non-Gorenstein-ness).

Algorithm

Aim: Compute $S_1(N, \epsilon, \mathbb{F}_p)$.

Problem: Modular symbols only work for weight ≥ 2.

Algorithm due to Edixhoven based on exact sequence

$$0 \longrightarrow S_1(N, \epsilon, \mathbb{F}_p) \xrightarrow{F} S_p(N, \epsilon, \mathbb{F}_p) \xrightarrow{\Theta} S_{p+2}(N, \epsilon, \mathbb{F}_p)$$

with $F =$ Frobenius $\sum a_n q^n \mapsto \sum a_n q^{np}$

and $\Theta =$ derivation $\sum a_n q^n \mapsto \sum n a_n q^n$.

Implementation based on William Stein's modular symbols over $\mathbb{F}_p$.

Example

```
> AttachSpec("PATH1/ArtinAlgebras.spec");
> AttachSpec("PATH2/Weight1.spec");
> w := Create(1429,2);
> SystemsOfEigenvalues(~w);
> ef := EigenformsWt1(w); #ef;
2
```
[i.e. two weight one forms over $\mathbb{F}_2$ in level 1429]

```
> Group(ef[2]);
SL(2,8) SL(2,8)
```
[image of Galois rep. of 2nd form is $\mathrm{SL}_2(\mathbb{F}_8)$]

```
> H := HeckeAlgebra(ef[2]); H;
Matrix Algebra of degree 2 with 2 generators over GF(2^3)
> Coefficient(ef[2],11);
$.1
```
[generator of $\mathbb{F}_8$]