
MAGMA **package** HeckeAlgebra

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Hecke algebras over \mathbb{F}_p

Used and developed together with Lloyd Kilford.

Aims:

- (I) Computation of Hecke algebras of modular forms over \mathbb{F}_p in the following cases:
 - all eigenforms for a given level N , weight $k \geq 2$ and Dirichlet character $\epsilon : (\mathbb{Z}/N\mathbb{Z})^\times \rightarrow \overline{\mathbb{F}}_p^\times$,
 - dihedral modular forms coming from the Hilbert class field of a quadratic number field,
 - icosahedral modular forms (only in characteristic $p = 2$) given by an A_5 -polynomial.
- (II) Compute local properties of these algebras, e.g. the Gorenstein property.

First example: $N = 431, k = 2, p = 2$

```
> AttachSpec("PATH1/ArtinAlgebras.spec");  
> AttachSpec("PATH2/HeckeAlgebra.spec");  
  
> ha := HeckeAlgebras(DirichletGroup(431,GF(2)).1,2);  
> HeckeAlgebraLaTeX(mf,'table.tex');
```

Get this table:

Level	Wt	ResD	Dim	EmbDim	NilO	GorDef	#Ops	#(p<HB)	Gp
431	2	1	4	3	1	2	6	20	
431	2	3	1	0	0	0	6	20	
431	2	3	3	1	2	0	6	20	
431	2	4	2	1	1	0	6	20	
431	2	6	1	0	0	0	6	20	
431	2	6	1	0	0	0	6	20	

Second example

```
> ha := [ ];  
> dih := DihedralForms(431 : ListOfPrimes := [2,11]);  
> for f in dih do ha := ha cat HeckeAlgebras(f); end for;  
> f := A5Form( $x^5 - x^4 - 780x^3 - 1795x^2 + 3106x + 344$ );  
> ha := ha cat HeckeAlgebras(f);  
> HeckeAlgebraLaTeX(ha, "table.tex");
```

Get this table:

Level	Wt	ResD	Dim	EmbDim	NilO	GorDef	#Ops	#(p<HB)	Gp
431	2	1	4	3	1	2	6	20	D_3
431	11	3	4	3	1	2	5	77	D_7
1951	2	4	3	1	2	0	3	66	A_5
1951	2	4	6	2	3	0	3	66	A_5