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# Fast Hecke Operator

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# Description and idea

Aim: Fast computation of Hecke operator  $T_p$  on weight 2 modular symbols on  $\Gamma_0(N)$  for  $N$  'small' and  $p$  'large'.

Idea: The Hecke operator  $T_p$  on the Manin presentation of modular symbols is described by Heilbronn matrices or similar sets of matrices (e.g. due to Merel).

These sets of matrices have  $O(p)$  elements.

In weight 2: only need matrices modulo  $N$

Note:  $\#M_2(\mathbb{Z}/N\mathbb{Z})_{\det=p} = O(N^3)$ .

So, when  $p > N^3$ , gain speed and memory by computing Heilbronn matrices directly modulo  $N$ .

Also, use trick by Basmaji for enumerating Heilbronn matrices quickly.

# Performance

Fast Hecke operator on level  $N = 360$ , weight  $k = 2$ .

CPU: AMD Athlon64X2

RAM: 8 GB

|           |           |           |           |        |           |
|-----------|-----------|-----------|-----------|--------|-----------|
| NextPrime | 10        | $10^2$    | $10^3$    | $10^4$ | $10^5$    |
| Time      | 47s       | 48s       | 48s       | 47s    | 48s       |
| NextPrime | $10^6$    | $10^7$    | $10^8$    | $10^9$ | $10^{10}$ |
| Time      | 48s       | 49s       | 60s       | 160s   | 19min     |
| NextPrime | $10^{11}$ | $10^{12}$ | $10^{13}$ |        |           |
| Time      | 3h        | 31h       | 13.5d     |        |           |

# Example

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```
> Attach("EssenModular.m");  
> M := ModularSymbols(360, 2, 1);  
> C := CuspidalSubspace(M);  
> time T := EssenHeckeOperator(C, NextPrime(10^9));  
Time: 160.850
```