MAGMA **package** `ArtinAlgebras`

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Artin algebras

Some tools for finite dimensional commutative algebras over fields, e.g. Hecke algebras (as matrix algebras).

Always have the local decomposition

$$A \cong \prod_{\mathfrak{m} \lhd A \text{ max.}} A_{\mathfrak{m}}$$

with $A_{\mathfrak{m}}$ the localisation of A at $\mathfrak{m}$.

This corresponds to idempotents (think of $(1, 0, \ldots, 0)$).

Always isomorphic to an affine algebra

$$A \cong k[X_1, \ldots, X_n]/(f_1, \ldots, f_m)$$

for some $m \geq n$.

The package

Main functions included in `ArtinAlgebras`:

- Given A (as a matrix algebra), compute all $A_{\mathfrak{m}}$.
- Compute the corresponding idempotents.
- If A is local k-algebra with residue field k, compute n, m and $f_1, \ldots, f_m$ such that

$$A \cong k[X_1, \ldots, X_n]/(f_1, \ldots, f_m).$$

  Hence, can save an algebra (up to isomorphism) by saving these data. Some 'trivial relations' can be omitted.
- Compute local properties of A, e.g. the Gorenstein defect.

Example

```
> AttachSpec("PATH/ArtinAlgebras.spec");
> M := CuspidalSubspace(ModularSymbols(431,2,GF(2),1));
> L := [HeckeOperator(M,n) : n in [1..50]];
> A := MatrixAlgebra(L);
> D := Decomposition(A);
> [AffineAlgebraTup(BaseChange(A,d)): d in D];
```

$[<$ Finite field of size $2, 3, 1, <>>,$
$<$ Finite field of size $2^4, 1, 1, <>>,$
$<$ Finite field of size $2^3, 1, 2, <>>,$
$<$ Finite field of size $2^3, 0, 0, <>>,$
$<$ Finite field of size $2^6, 0, 0, <>>,$
$<$ Finite field of size $2^6, 0, 0, <>>]$

```
> [GorensteinDefect(BaseChange(A,d)): d in D];
```

$[2, 0, 0, 0, 0, 0]$